



# Multi-Modal Image Processing with Applications to Art Investigation and Beyond

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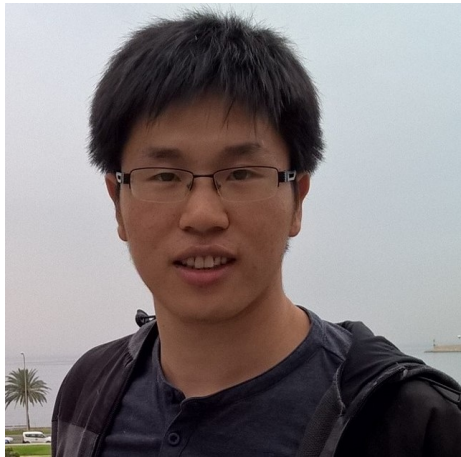
# Collaborators



Ingrid Daubechies  
Duke U.



Bruno Cornelis  
VUB



Pingfan Song  
UCL



Joao Mota  
Heriot Watt U.



Nikos Deligiannis  
VUB

# Multi-Modal Data Processing in Healthcare

## Medical Imaging

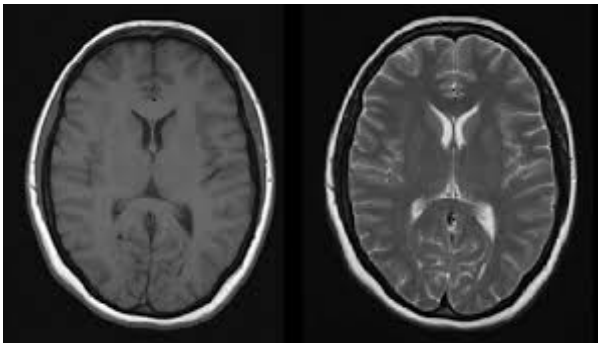


## Emerging questions

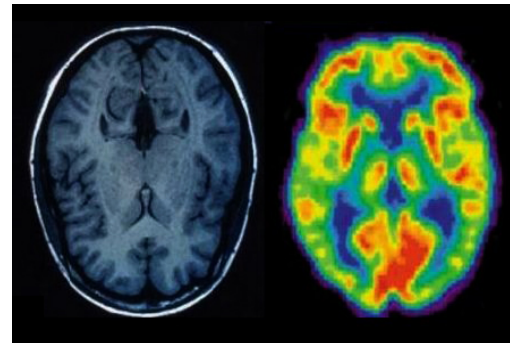
The questions that arise in medical imaging include:

- How to trade-off acquisition resolution across the various imaging modalities?
- How to analyse multiple complementary image modalities?

T1 and T2

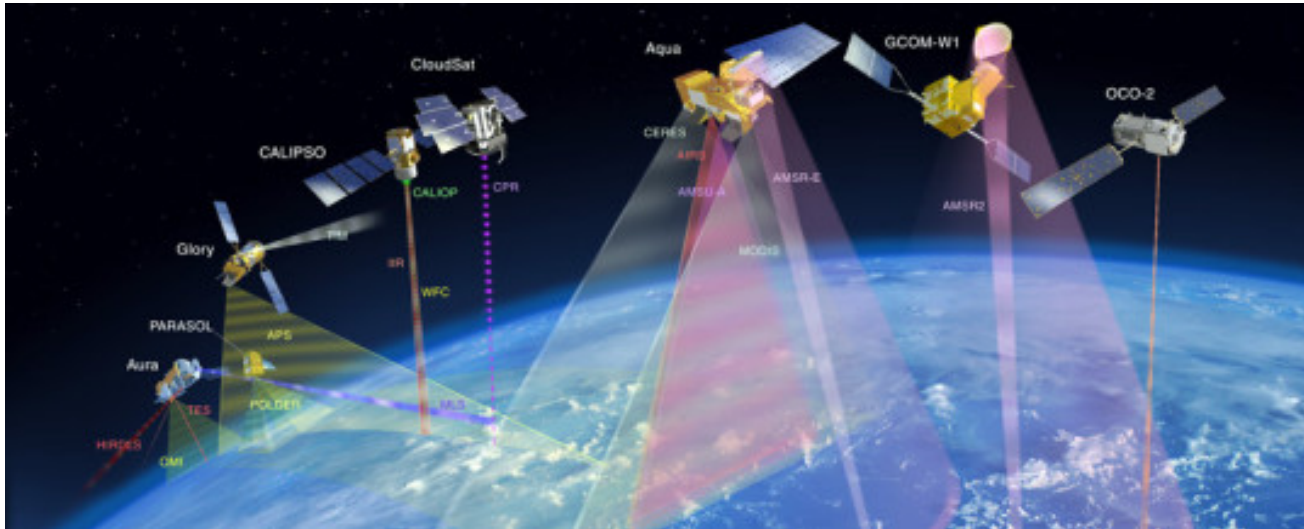


MRI and PET



# Multi-Modal Data Processing in Engineering

## Remote Sensing

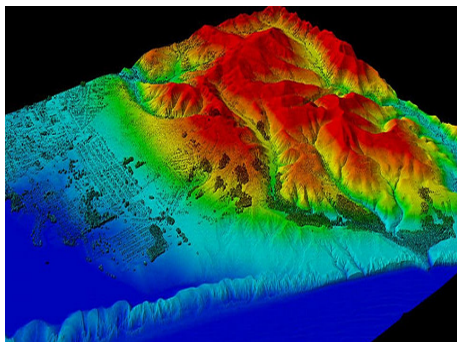


## Emerging questions

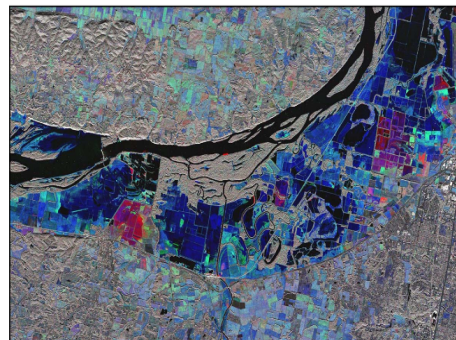
The questions that arise in remote sensing also include:

- How to trade-off acquisition resolution across the various imaging modalities?
- How to analyse multiple complementary image modalities?

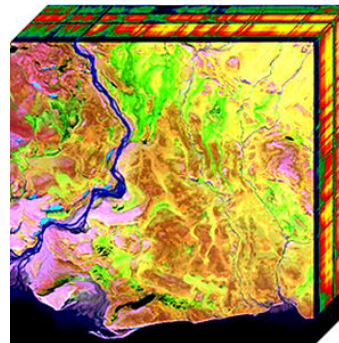
LIDAR Data



SAR Data

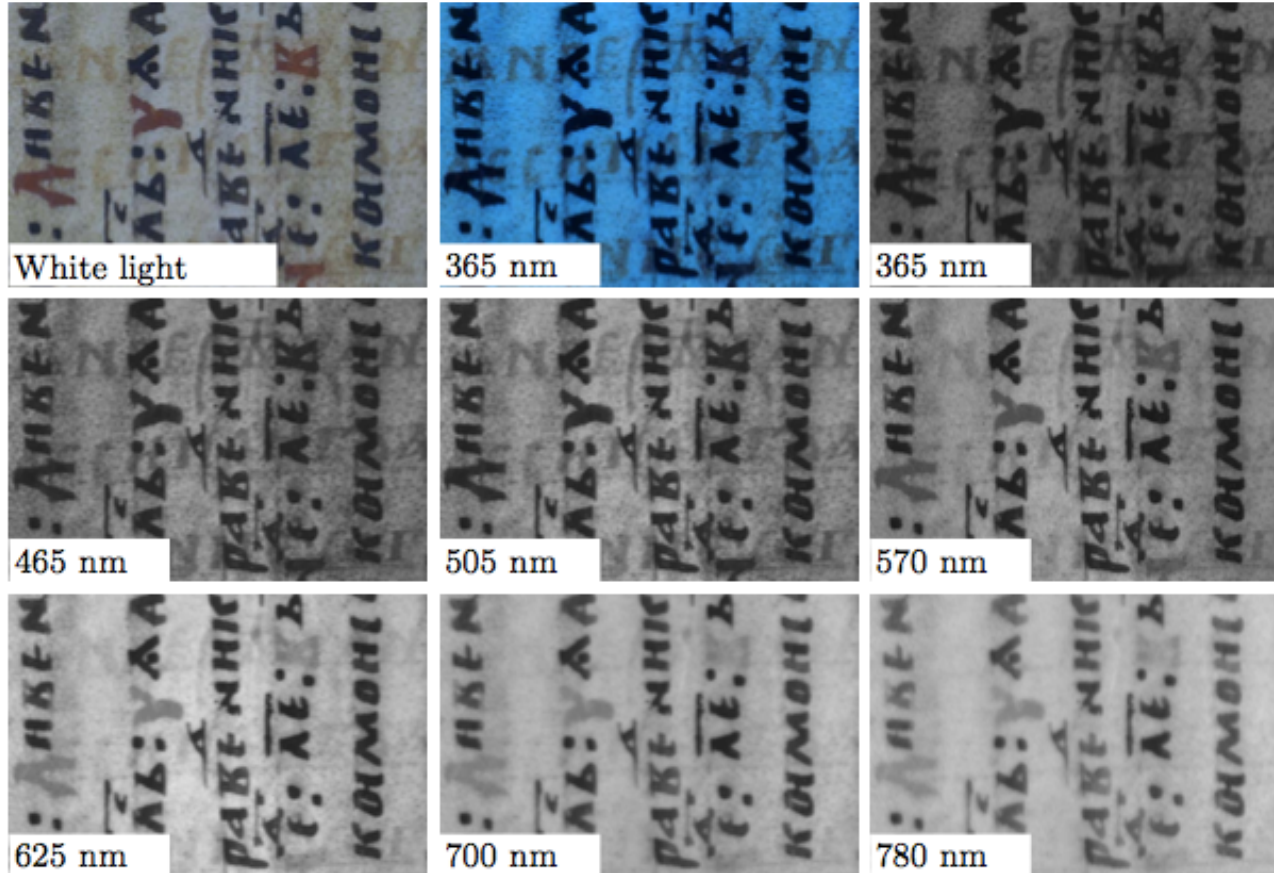


Hyper-Spectral Data



# Multi-Modal Data Processing in Arts and Humanities

## Palimpsests in Cultural Heritage and Archeology



Palimpsest contains a Cyrillic overwriting and partly Greek, partly Cyrillic underwritings, which have been washed off

## Emerging questions

- Common practice in medieval ecclesiastical circles to rub out an earlier piece of writing by means of washing or scraping the manuscript, in order to prepare it for a new text.
- Modern historians are usually more interested in older writings, so multi-modal data processing technology is needed to attempt to recover erased old texts.

# Multi-Modal Data Processing in Arts and Humanities

## Art Investigation, Preservation and Restoration

The Ghent Altarpiece -  
Visuals



The Ghent Altarpiece -  
X-Rays



## Emerging questions

Some tasks that arise in art investigation, restoration and preservation include:

- The separation of paintings onto different layers for technical study purposes.
- The identification of areas associated with degradation / restoration.

The imaging modalities used in art investigation include macrophotography, X-radiography, hyperspectral imaging, infrared imaging, X-ray fluorescence (XRF) mapping

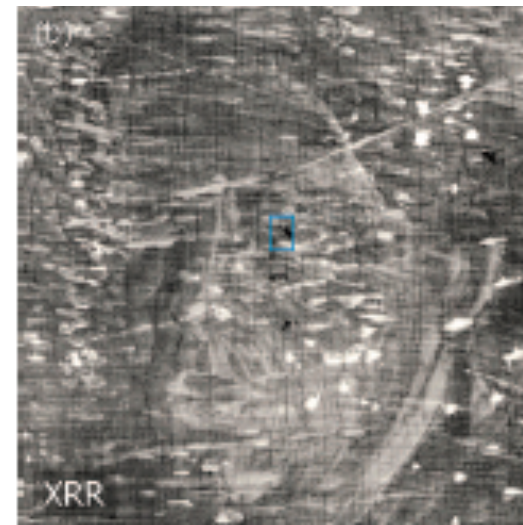
# Multi-Modal Data Processing in Arts and Humanities

Vincent van Gogh

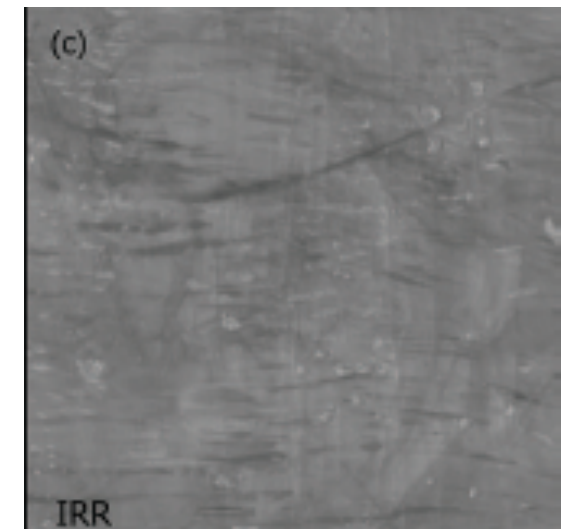
Patch of Grass, Paris, Apr-June 1887



X-ray radiation transmission radiograph (XRR)



Infrared reflectograph (IRR)



Dik et al. Visualization of a Lost Painting by Vincent van Gogh Using Synchrotron Radiation Based X-ray Fluorescence Elemental Mapping. *Anal. Chem.* 2008, 80, 6436–6442

# Outline

- i. Parsimonious Representations for Unimodal Data Processing
- ii. Joint Parsimonious Representations for Multimodal Data Processing
- iii. Multimodal Data Aided Processing
  - a. Image separation aided by multimodal data
  - b. Image super-resolution aided by multimodal data
- iv. Concluding Remarks and Directions



# Outline

- i. **Parsimonious Representations for Unimodal Data Processing**
- ii. Joint Parsimonious Representations for Multimodal Data Processing
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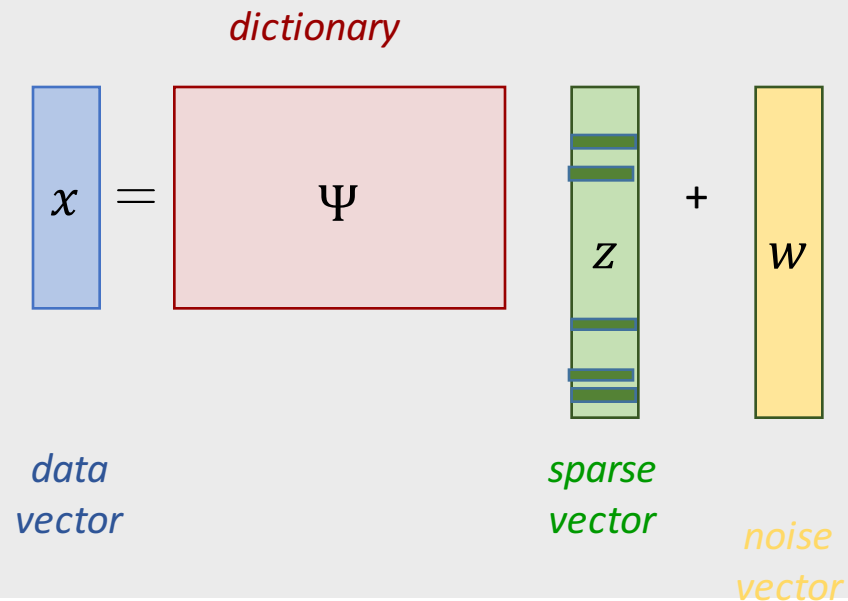
# Sparse Representations for Data Processing

## Parsimonious representations

The data vector  $x \in \mathbb{R}^n$  can be represented in terms of a sparse vector  $z \in \mathbb{R}^m$  as follows:

$$x = \Psi z + w$$

where  $\Psi \in \mathbb{R}^{n \times m}$  is a dictionary such as a wavelet basis or a learnt one.



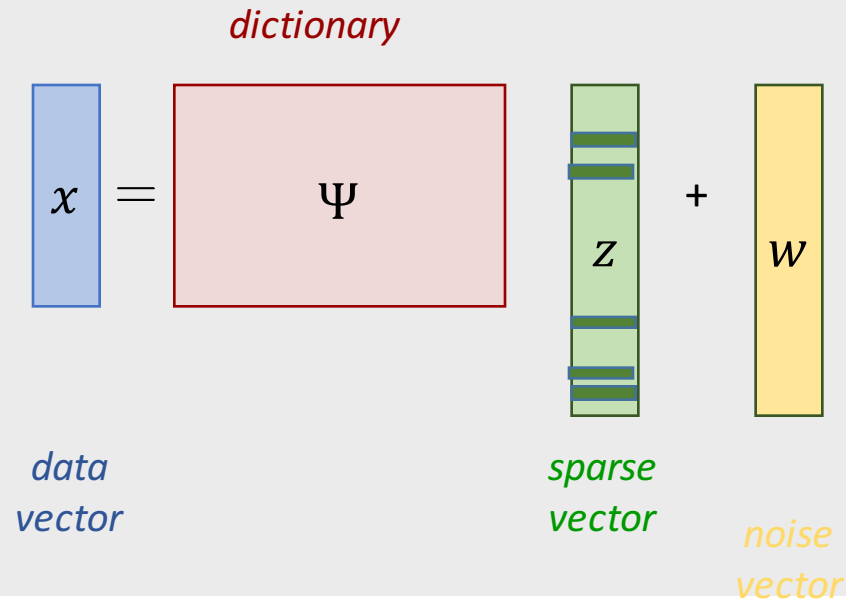
# Sparse Representations for Data Processing

## Parsimonious representations

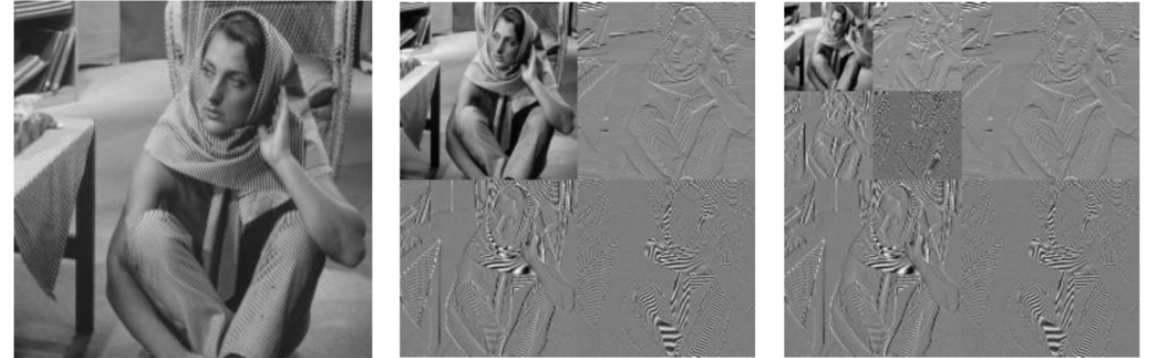
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## Wavelet representations



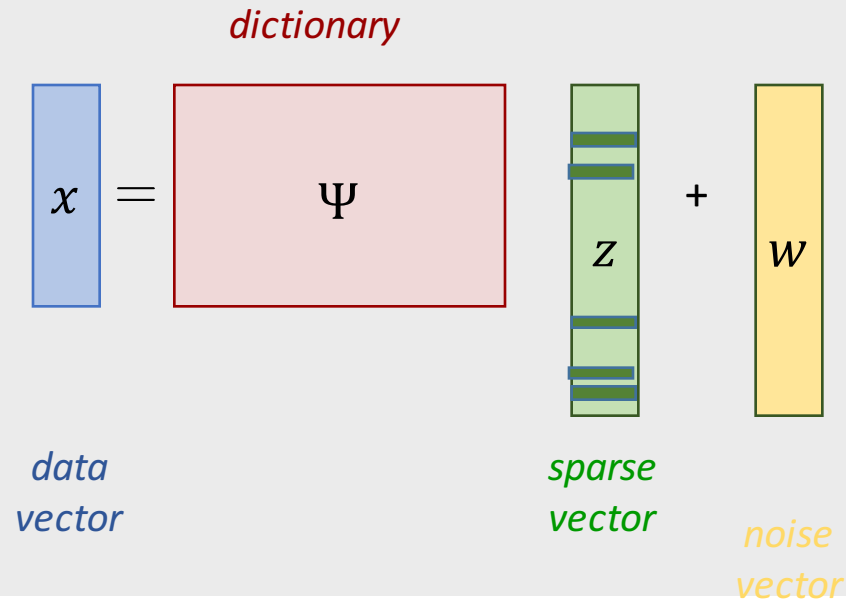
# Sparse Representations for Data Processing

## Parsimonious representations

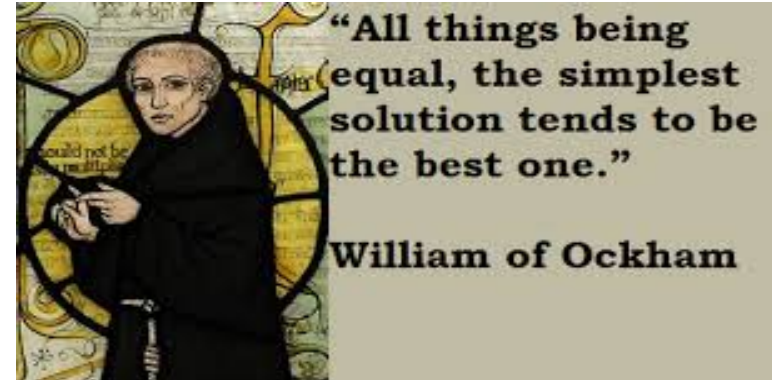
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## Occam's Razor



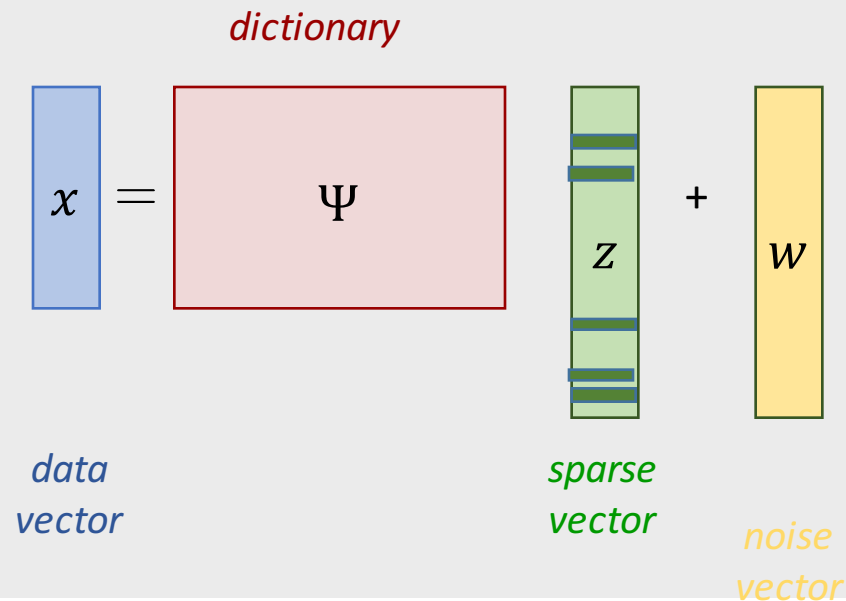
# Sparse Representations for Data Processing

## Parsimonious representations

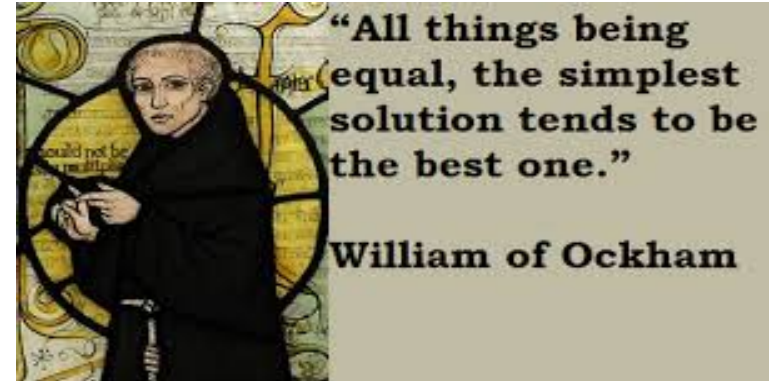
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## Occam's Razor



## Applications

Sparse representations have had implications in various problems such as:

1. Compressive sensing
2. Image in-painting, denoising, deblurring
3. Image super-resolution
4. Source separation/de-mixing

# The Compressive Sensing Problem

## Signal Sensing

The measurement vector is generated from the signal vector as follows:

$$y = \Phi x = \Phi \Psi z$$

where  $\Phi$  is a “wide” measurement matrix.

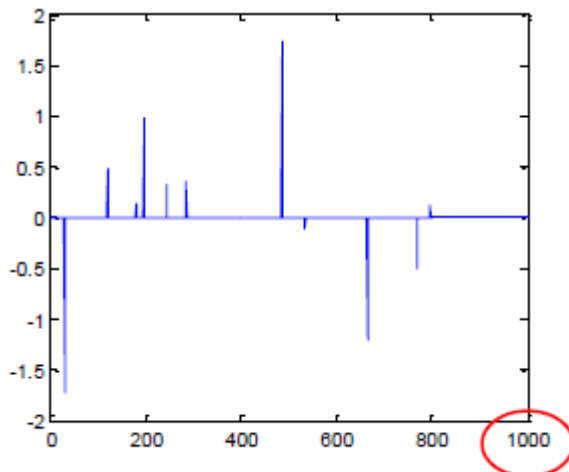
## Signal Reconstruction

The signal sparse representation vector can be recovered from the measurement vector as follows:

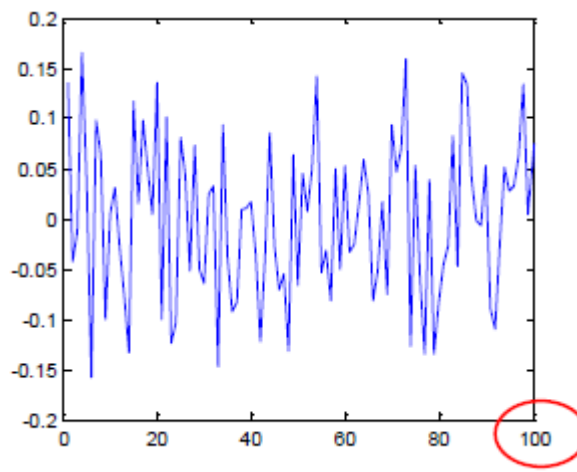
$$\hat{z} = \arg \min_z \|z\|_1 \text{ subject to } y = \Phi \Psi z$$

Optimization- and greedy-based algorithms can be used to reconstruct the signal vector from the measurement vector.

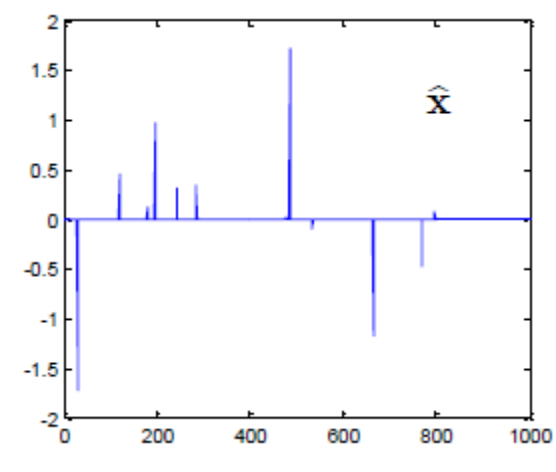
Sparse Vector ( $z$ )



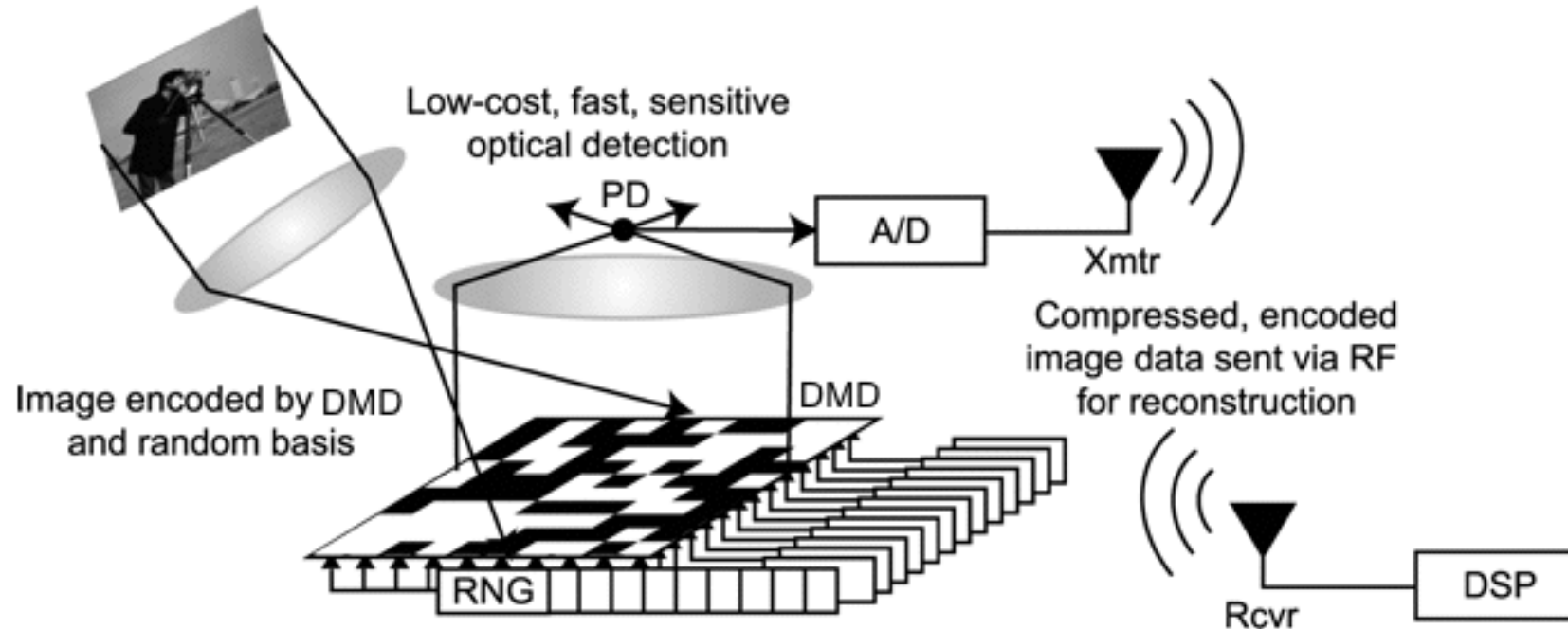
Measured Vector ( $y$ )



Recovered Sparse Vector



# The Compressive Sensing Problem: The Single-Pixel Camera



# Image De-Noising, De-Blurring and In-Painting

## De-Noising

Noisy Image



De-Noised Image



## De-Blurring

Blurred Image



De-Blurred Image



## In-Painting

Original Image



New Image



## Angle-of-Attack

### Model

One postulates that the true image admits a sparse representation in some dictionary.

### Algorithm

One then obtains the sparse represent. associated with the image as well as the dictionary given the noisy / blurred / in-painted image.



# Image De-Noising

## De-noising model

One observes a noisy version  $y_i$  of image (patches)  $x_i$ :

$$y_i = x_i + w_i, \quad \forall i$$

The image (patches)  $x_i$  obey a sparse representation  $z_i$  in a dictionary  $D$ :

$$x_i = Dz_i, \quad \forall i$$

## Sparse representations based de-noising

This problem can be addressed using sparse representations whereby the de-noised image is generated from the noisy image as follows:

$$\min_{D, z_i} \sum_i \|y_i - Dz_i\|_2^2 + \|z_i\|_1 \quad \longrightarrow \quad \hat{x}_i = D\hat{z}_i, \quad \forall i$$

## Original Noisy Image



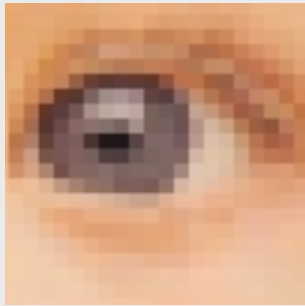
## De-noised Image



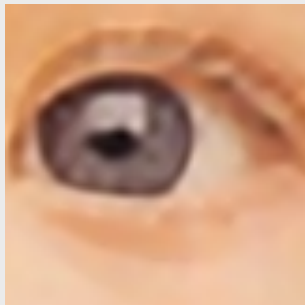
# Image Super-Resolution (SR)

## Super-Resolution Problem

Low-resolution Image



High-resolution Image



## Angle-of-Attack

### Model

One postulates that both the HR and the LR images admit a sparse representation in HR and LR dictionaries.

### Algorithm

One then obtains the HR image from the LR image by determining the sparse representation associated with the images as well as the HR and LR dictionaries.

# Image Super-Resolution

## Super-resolution model

One postulates that HR patches  $x_i^{HR}$  and LR patches  $x_i^{LR}$  admit a common sparse representation  $z_i$  in HR and LR dictionaries  $D^{HR}$  and  $D^{LR}$ :

$$x_i^{HR} = D^{HR} z_i, \forall i$$

$$x_i^{LR} = D^{LR} z_i, \forall i$$

## Sparse representations based super-resolution

This problem can be addressed using sparse representations whereby the HR image is generated from the LR image as follows:

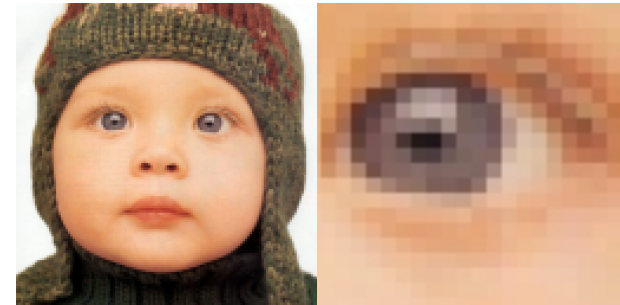
Training:

$$\min_{D^{HR}, D^{LR}, z_i} \sum_i \|x_i^{HR} - D^{HR} z_i\|_2^2 + \|x_i^{LR} - D^{LR} z_i\|_2^2 + \lambda \cdot \|z_i\|_1$$

Testing:

$$\hat{z}_i = \operatorname{argmin}_{z_i} \|x_i^{LR} - D^{LR} z_i\|_2^2 + \lambda \cdot \|z_i\|_1 \quad \longrightarrow \quad \hat{x}_i^{HR} = D^{HR} \hat{z}_i$$

## Low-resolution Image



## High-resolution Image



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# Joint Sparse Representations for Multi-Modal Data

## Wishlist

1. Model to represent accurately each individual image modality;
2. Model to connect the various image modalities;
3. Model to be readily learnt from data using simple algorithms;
4. Model to lead to simple multi-modal processing algorithms.

## Joint Parsimonious Representations

Each individual image modalities admit sparse representations in a dictionary.

The various image modalities are connected via sparse representations.

$$\begin{aligned} x_1 &= \Phi^c z^c + \Phi z_1 && \text{data modality 1} \\ x_2 &= \Psi^c z^c + \Psi z_2 && \text{data modality 2} \end{aligned}$$

*Common Components*      *Innovation Components*

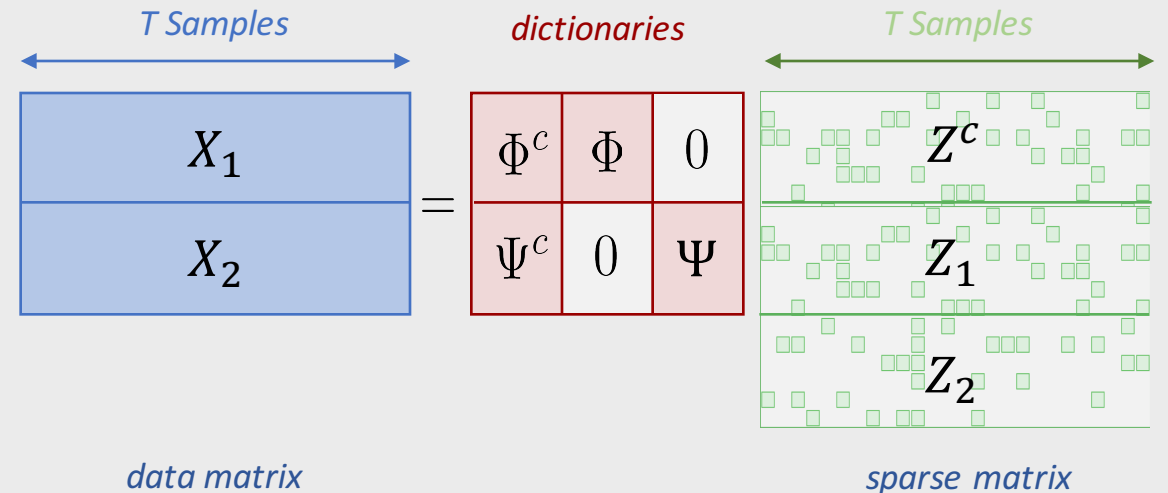
# Joint Sparse Representations for Multi-Modal Data

## Wishlist

1. Model to represent accurately each individual image modality;
2. Model to connect the various image modalities;
3. Model to be readily learnt from data using simple algorithms;
4. Model to lead to simple multi-modal processing algorithms.

## Learning, Analysis and Processing Algorithms

Our model can also be readily learnt using matrix factorization techniques.



Our model also leads to simple multi-modal image processing algorithms that exploit the joint sparse representations.

# Joint Sparse Representations for Multi-Modal Data

## Wishlist

1. Model to represent accurately each individual image modality;
2. Model to connect the various image modalities;
3. Model to be readily learnt from data using simple algorithms;
4. Model to lead to simple multi-modal processing algorithms.

## Coupled Dictionary Learning Algorithm

$$\min_{\substack{\Phi^c, \Phi, \Psi^c, \Psi \\ Z^c, Z_1, Z_2}} \|X_1 - \Phi^c Z^c - \Phi Z_1\|_F^2 + \|X_2 - \Psi^c Z^c - \Psi Z_2\|_F^2$$

$$\text{s. t. } \text{card}(Z^c(i)) \leq s_c, i = 1, \dots, T$$

$$\text{card}(Z_1(i)) \leq s_1, i = 1, \dots, T$$

$$\text{card}(Z_2(i)) \leq s_2, i = 1, \dots, T$$



Learn dictionaries by alternating between:

1. Learning the sparse representations given the dictionaries (sparse coding step)
2. Learning the dictionaries given the sparse representations (dictionary update step)

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# Multi-Modal Data Aided Image Separation

## Problem

This problem involves separating the super-position of the x-rays given the visuals.

## Model<sub>coupling</sub>

$$y = \Psi^c z$$

*Visual*

$$x = \Phi^c z + \Phi v$$

*X-Ray*



*Visual Rear Panel*



*Visual Front Panel*



*Mixed X-Ray*

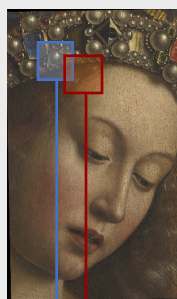
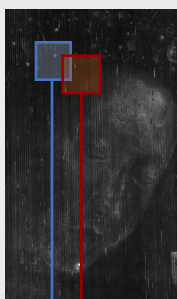
# Multi-Modal Data Aided Image Separation

## Learning Phase

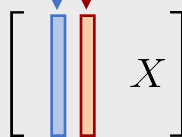
The goal is to learn the joint parsimonious model from available data.

### Algorithm

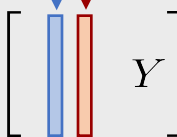
*X-Ray*



*Visible*



$X$



$Y$

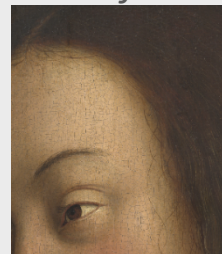
$$\begin{aligned} & \underset{\substack{\Psi^c, \Phi^c, \Phi \\ Z, V}}{\text{minimize}} && \|Y - \Psi^c Z\|_F^2 + \|X - \Phi^c Z - \Phi V\|_F^2 \\ & \text{subject to} && \text{card}(Z_i) \leq s_z, \quad i = 1, \dots, T \\ & && \text{card}(V_i) \leq s_v, \quad i = 1, \dots, T \end{aligned}$$

## Processing Phase

The goal is to unmix the x-rays given the x-ray mixture and the visuals.

### Algorithm

*visual front*



$$y_1 = \Psi^c z_1$$

*visual back*



$$y_2 = \Psi^c z_2$$

*mixed x-ray*

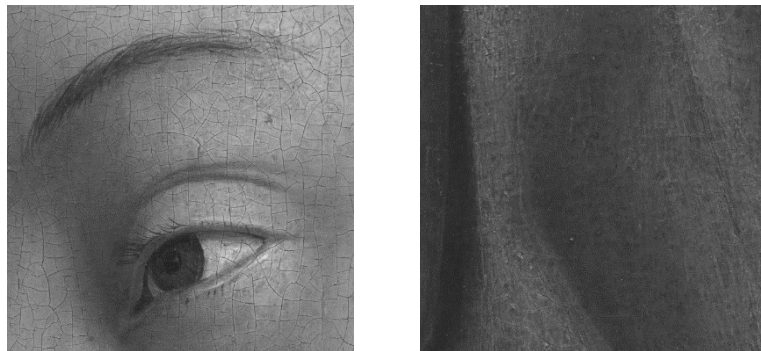


$$x = \Phi^c(z_1 + z_2) + 2\Phi v$$

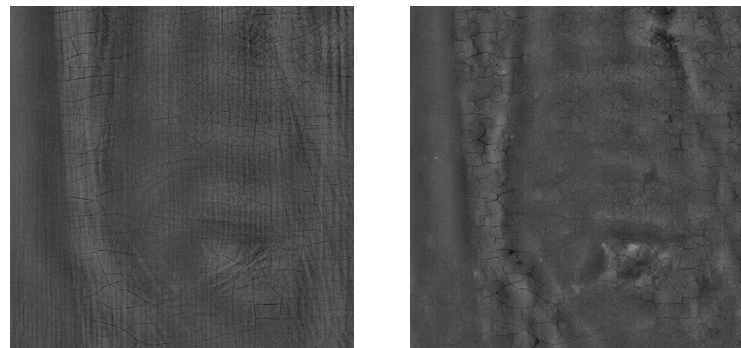
$$\begin{aligned} & \underset{z_1, z_2, v}{\text{minimize}} && \|z_1\|_1 + \|z_2\|_1 + \|v\|_1 \\ & \text{subject to} && x = \Phi^c(z_1 + z_2) + 2\Phi v \\ & && y_1 = \Psi^c z_1 \\ & && y_2 = \Psi^c z_2 \end{aligned}$$

# Multi-Modal Data Aided Image Separation

*visuals in grayscale*

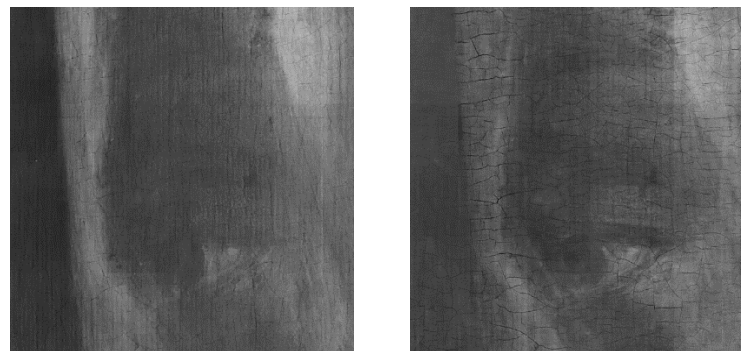


*reconstructed x-rays*

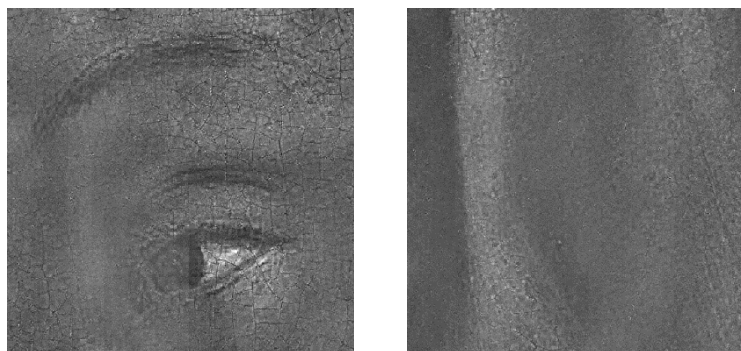


MCA

*mixed x-ray*



multiscale  
MCA w/KSVD



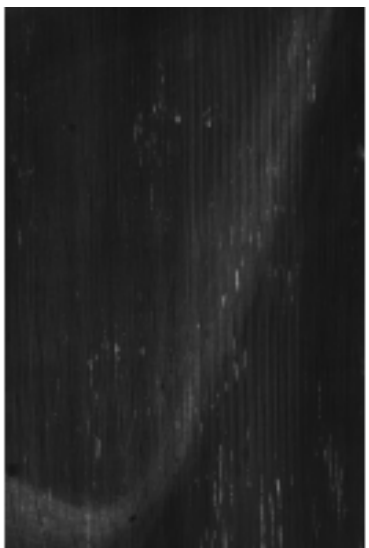
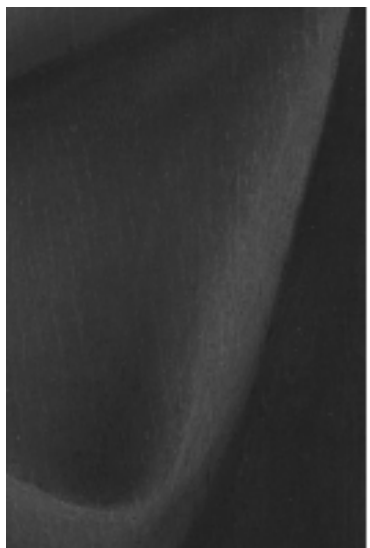
**Ours**

# Multi-Modal Data Aided Image Separation

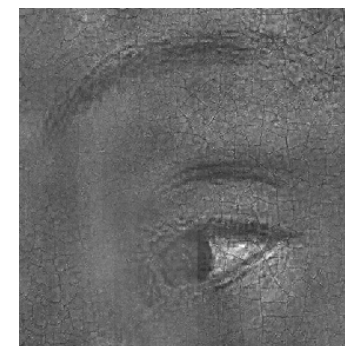
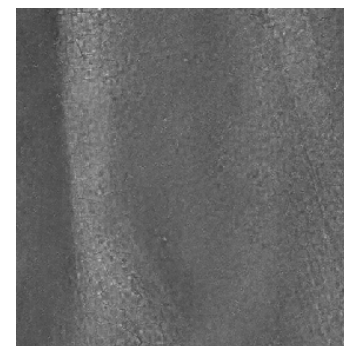
*Visual*

*X-Rays*

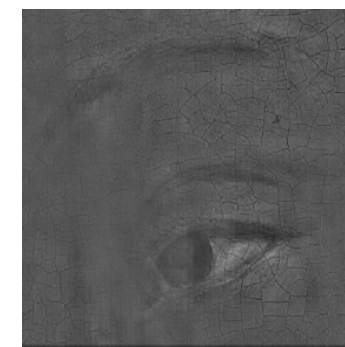
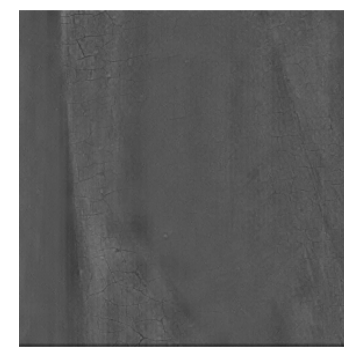
*Crack Mask*



*Mixed  
X-Rays*



*Separation  
based on CDL*



*Separation  
based on  
Weighted CDL*

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# Multi-Modal Data Aided Super-Resolution

## Problem

This problem involves producing a high-resolution (HR) image from a low-resolution (LR) one of the same scene, by leveraging the presence of other images associated with the scene.

## Model

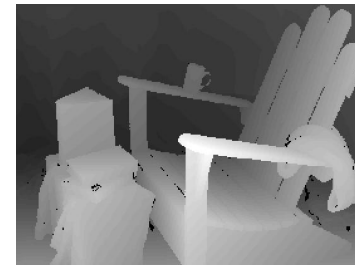
$$x^{hr} = \Psi_c^{hr} z_c + \Psi^{hr} u \quad \text{--- HR image of interest}$$

$$x^{lr} = \Psi_c^{lr} z_c + \Psi^{lr} u \quad \text{--- LR image of interest}$$

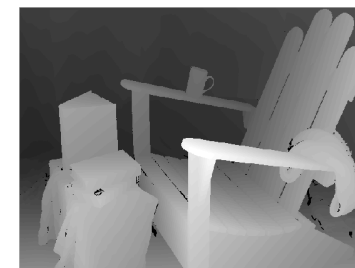
$$y^{hr} = \Phi_c^{hr} z_c + \Phi^{hr} v \quad \text{--- another HR image}$$

*coupling between HR and LR image*

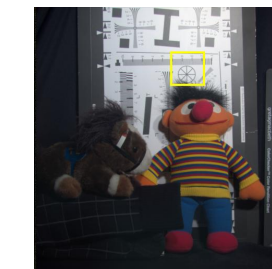
LR Image



HR Image



HR Side Information



# Multi-Modal Data Aided Super-Resolution

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## Model

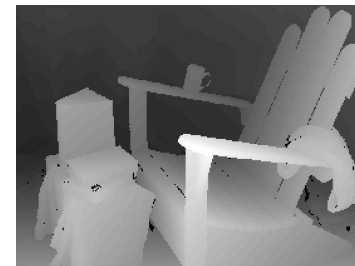
$$x^{hr} = \Psi_c^{hr} z_c + \Psi^{hr} u \quad \text{--- HR image of interest}$$

$$x^{lr} = \Psi_c^{lr} z_c + \Psi^{lr} u \quad \text{--- LR image of interest}$$

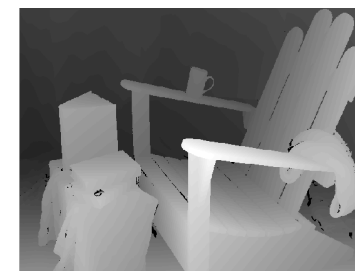
$$y^{hr} = \Phi_c^{hr} z_c + \Phi^{hr} v \quad \text{--- another HR image}$$

*coupling between modalities*

LR Image



HR Image



HR Side Information



# Multi-Modal Data Aided Super-Resolution

## Problem

This problem involves producing a high-resolution (HR) image from a low-resolution (LR) one of the same scene, by leveraging the presence of other images associated with the scene.

## Model

$$x^{hr} = \Psi_c^{hr} z_c + \Psi^{hr} u \quad \text{--- HR image of interest}$$

$$x^{lr} = \Psi_c^{lr} z_c + \Psi^{lr} u \quad \text{--- LR image of interest}$$

$$y^{hr} = \Phi_c^{hr} z_c + \Phi^{hr} v \quad \text{--- another HR image}$$

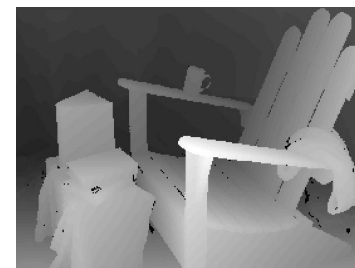
*coupling between modalities*

Training Phase

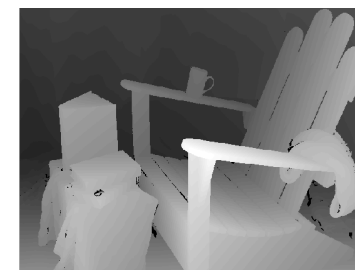


Processing Phase

LR Image



HR Image



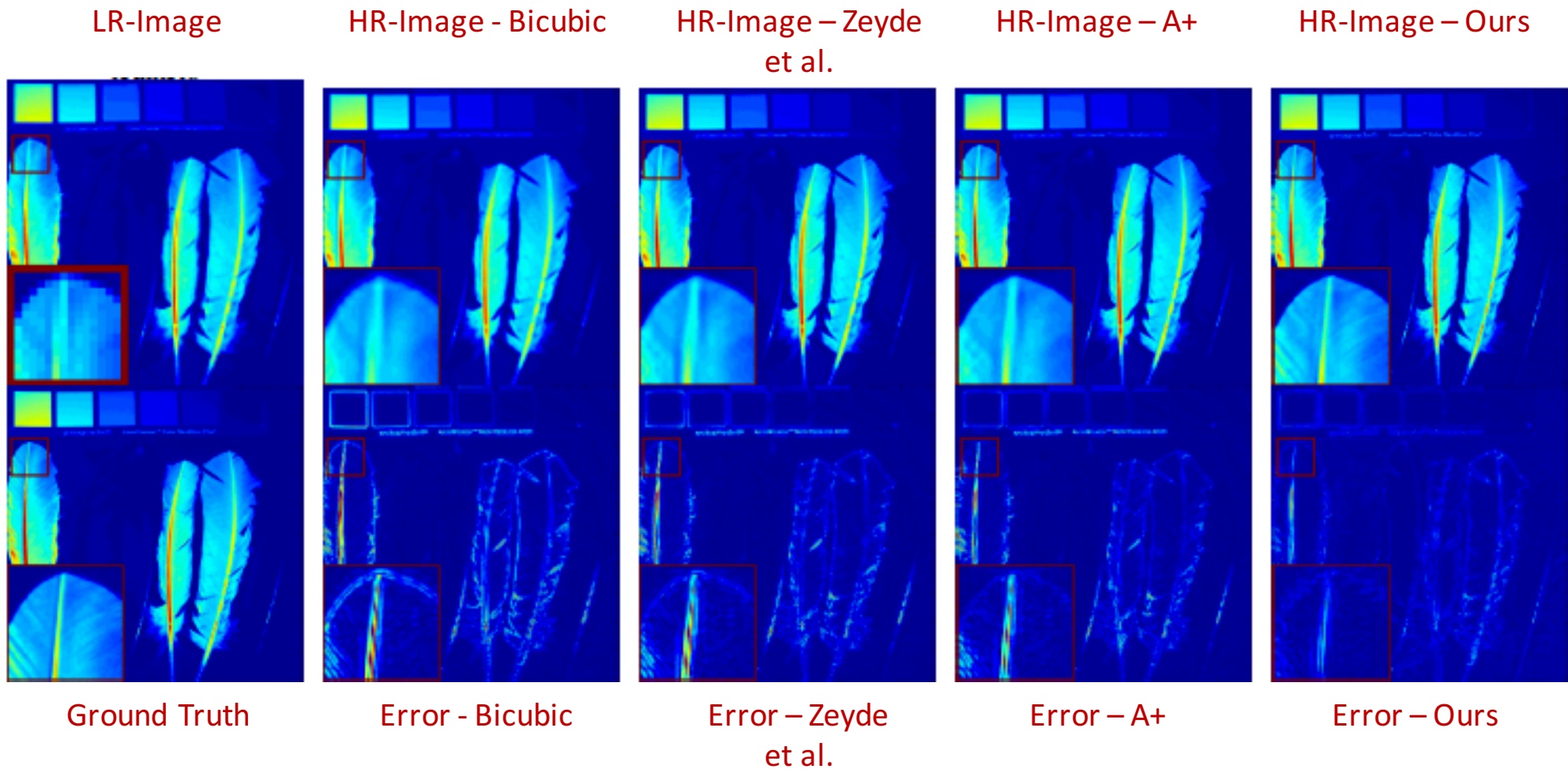
HR Side Information





# Multi-Modal Data Aided Super-Resolution

Super-resolving hyper-spectral images with the aid of RGB images



# Multi-Modal Data Aided Super-Resolution

Super-resolving infrared images with the aid of RGB images

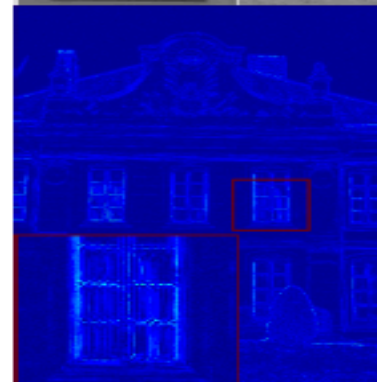
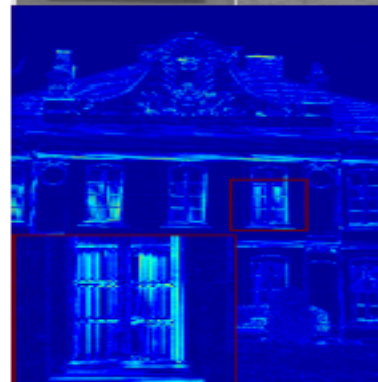
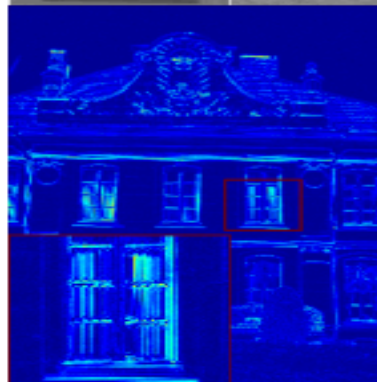
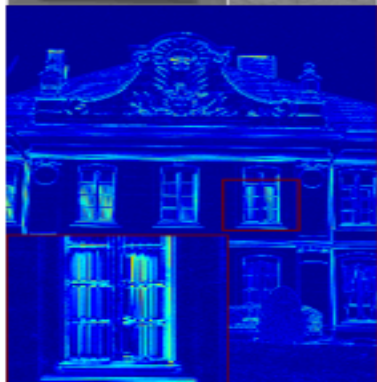
LR-Image

HR-Image - Bicubic

HR-Image - Zeyde  
et al.

HR-Image - A+

HR-Image - Ours



Ground Truth

Error - Bicubic

Error - Zeyde  
et al.

Error - A+

Error - Ours

# Outline

- i. Parsimonious Representations for Unimodal Data Processing
- ii. Joint Parsimonious Representations for Multimodal Data Processing
- iii. Multimodal Data Aided Processing**
  - a. Image separation aided by multimodal data
  - b. Image super-resolution aided by multimodal data
- iv. Concluding Remarks and Directions

# Concluding Remarks and Directions

- i. Joint sparse representations induced by coupled dictionaries can also address emerging multi-modal data processing problems.
- ii. A number of applications have been demonstrated in the context of art-investigation and beyond.
- iii. The techniques can be used to address various other multi-modal imaging processing tasks and applications.